Monday, September 21, 2015

Page 390

Problem 43

Problem. Find the indefinite integral $\int \cosh 2x \, dx$.

Solution. Let $u = 2x$ and $du = 2\,dx$. Then

$$\begin{aligned}\int \cosh 2x \, dx &= \frac{1}{2}\int 2\cosh 2x \, dx \\ &= \frac{1}{2}\int \cosh u \, du \\ &= \frac{1}{2}\sinh u + C \\ &= \frac{1}{2}\sinh 2x + C.\end{aligned}$$

Problem 47

Problem. Find the indefinite integral $\int \cosh^2(x-1)\sinh(x-1)\, dx$.

Solution. Let $u = x - 1$ and $du = dx$. Then

$$\int \cosh^2(x-1)\sinh(x-1)\, dx = \int \cosh^2 u \sinh u \, du.$$

Now let $v = \cosh u$ and $dv = \sinh u \, du$. Then

$$\begin{aligned}\int \cosh^2 u \sinh u \, du &= \int v^2 \, dv \\ &= \frac{1}{3}v^3 + C \\ &= \frac{1}{3}\cosh^3 u + C \\ &= \frac{1}{3}\cosh^3(x-1) + C.\end{aligned}$$

Problem 55

Problem. Evaluate the integral $\int_0^{\ln 2} \tanh x \, dx$.

Solution. Use the fact that $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ and the fact that the numerator is the derivative of the denominator. Then

$$\begin{aligned}
\int_0^{\ln 2} \tanh x \, dx &= \int_0^{\ln 2} \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx \\
&= \left[\ln |e^x + e^{-x}|\right]_0^{\ln 2} \\
&= \ln |e^{\ln 2} + e^{-\ln 2}| - \ln |e^0 + e^{-0}| \\
&= \ln \left(2 + \frac{1}{2}\right) - \ln 2 \\
&= \ln 1.25.
\end{aligned}$$

Problem 57

Problem. Evaluate the integral $\displaystyle\int_0^4 \frac{1}{25 - x^2} \, dx$.

Solution. Let $x = 5u$ and $dx = 5 \, du$. Note that $u(0) = 0$ and $u(4) = \frac{4}{5}$. Then

$$\begin{aligned}
\int_0^4 \frac{1}{25 - x^2} \, dx &= \int_0^{4/5} \frac{5}{25 - 25u^2} \, du \\
&= \frac{1}{5} \int_0^{4/5} \frac{1}{1 - u^2} \, du \\
&= \frac{1}{5} \left[\frac{1}{2} \ln \left(\frac{1+u}{1-u}\right)\right]_0^{4/5} \\
&= \frac{1}{10} \left(\ln \left(\frac{1 + \frac{4}{5}}{1 - \frac{4}{5}}\right) - \ln \left(\frac{1}{1}\right)\right) \\
&= \frac{1}{10} \ln 9
\end{aligned}$$

Problem 59

Problem. Evaluate the integral $\displaystyle\int_0^{\sqrt{2}/4} \frac{2}{\sqrt{1 - 4x^2}} \, dx$.

Solution. Let $u = 2x$ and $du = 2\ dx$. Note that $u(0) = 0$ and $u(\sqrt{2}/4) = \sqrt{2}/2$. Then

$$\begin{aligned} \int_0^{\sqrt{2}/4} \frac{2}{\sqrt{1-4x^2}}\, dx &= \int_0^{\sqrt{2}/4} \frac{2}{\sqrt{1-(2x)^2}}\, dx \\ &= \int_0^{\sqrt{2}/2} \frac{1}{\sqrt{1-u^2}}\, du \\ &= [\arcsin u]_0^{\sqrt{2}/2} \\ &= \arcsin \frac{\sqrt{2}}{2} - \arcsin 0 \\ &= \frac{\pi}{4}. \end{aligned}$$

Problem 65

Problem. Find the derivative of the function $y = \cosh^{-1} 3x$.

Solution. Use the formula on page 389.

$$\begin{aligned} y' &= \frac{3}{\sqrt{(3x)^2 - 1}} \\ &= \frac{3}{\sqrt{9x^2 - 1}}. \end{aligned}$$

Problem 73

Problem. Find the derivative of the function $y = 2x \sinh^{-1}(2x) - \sqrt{1+4x^2}$.

Solution.

$$\begin{aligned} y' &= \left(2 \sinh^{-1}(2x) + 2x \cdot \frac{2}{\sqrt{(2x)^2+1}}\right) - \frac{1}{2} \cdot \frac{8x}{\sqrt{1+4x^2}} \\ &= 2 \sinh^{-1}(2x) + \frac{4x}{\sqrt{4x^2+1}} - \frac{4x}{\sqrt{4x^2+1}} \\ &= 2 \sinh^{-1}(2x). \end{aligned}$$

Problem 74

Problem. Find the derivative of the function $y = x \tanh^{-1} x + \ln \sqrt{1-x^2}$.

Solution.

$$\begin{aligned} y' &= \left(\tanh^{-1} x + x \cdot \frac{1}{1-x^2}\right) + \frac{\frac{1}{2} \cdot \frac{-2x}{\sqrt{1-x^2}}}{\sqrt{1-x^2}} \\ &= \tanh^{-1} x + \frac{x}{1-x^2} - \frac{x}{1-x^2} \\ &= \tanh^{-1} x. \end{aligned}$$

Problem 75

Problem. Find the indefinite integral $\int \frac{1}{3-9x^2}\,dx$ using the formulas from Theorem 5.20.

Solution. Factor 9 out of the denominator.

$$\int \frac{1}{3-9x^2}\,dx = \frac{1}{9}\int \frac{1}{\frac{1}{3}-x^2}\,dx.$$

Then use theorem 5.20 with $a = \frac{1}{\sqrt{3}}$.

$$\begin{aligned} \frac{1}{9}\int \frac{1}{\frac{1}{3}-x^2}\,dx &= \frac{1}{9} \cdot \frac{1}{2 \cdot \frac{1}{\sqrt{3}}} \ln\left|\frac{\frac{1}{\sqrt{3}}+x}{\frac{1}{\sqrt{3}}-x}\right| + C \\ &= \frac{1}{6\sqrt{3}} \ln\left|\frac{1+\sqrt{3}x}{1-\sqrt{3}x}\right| + C. \end{aligned}$$

Problem 77

Problem. Find the indefinite integral $\int \frac{1}{\sqrt{1+e^{2x}}}\,dx$ using the formulas from Theorem 5.20.

Solution. Let $u = e^x$ and $du = e^x\,dx$. Then make the substitution and use Theorem

5.20 with $a = 1$.

$$\begin{aligned}
\int \frac{1}{\sqrt{1+e^{2x}}}\,dx &= \int \frac{e^x}{e^x\sqrt{1+e^{2x}}}\,dx \\
&= \int \frac{1}{u\sqrt{1+u^2}}\,du \\
&= -\ln \frac{1+\sqrt{1+u^2}}{|u|} + C \\
&= -\ln \frac{1+\sqrt{1+e^{2x}}}{|e^x|} + C \\
&= -\ln\left(1+\sqrt{1+e^{2x}}\right) + \ln e^x + C \\
&= -\ln\left(1+\sqrt{1+e^{2x}}\right) + x + C \\
&= x - \ln\left(1+\sqrt{1+e^{2x}}\right) + C
\end{aligned}$$

This looks very different from the book's answer, but it is equivalent.

Problem 79

Problem. Find the indefinite integral $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$ using the formulas from Theorem 5.20.

Solution. Let $x = u^2$ and $dx = 2u\ du$. Then

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx &= \int \frac{2u}{\sqrt{u^2}\sqrt{1+u^2}}\,du \\
&= \int \frac{2u}{u\sqrt{1+u^2}}\,du \\
&= 2\int \frac{1}{\sqrt{1+u^2}}\,du \\
&= 2\ln\left(u+\sqrt{1+u^2}\right) \\
&= 2\ln\left(\sqrt{x}+\sqrt{1+x}\right)
\end{aligned}$$

Problem 81

Problem. Find the indefinite integral $\displaystyle\int \frac{-1}{4x - x^2}\,dx$ using the formulas from Theorem 5.20.

Solution. Complete the square in x.

$$\begin{aligned}
\int \frac{-1}{4x - x^2}\,dx &= \int \frac{1}{x^2 - 4x}\,dx \\
&= \int \frac{1}{(x^2 - 4x + 4) - 4}\,dx \\
&= \int \frac{1}{(x-2)^2 - 4}\,dx.
\end{aligned}$$

Let $u = x - 2$ and $du = dx$. Then

$$\begin{aligned}
\int \frac{1}{(x-2)^2 - 4}\,dx &= \int \frac{1}{u^2 - 4}\,du \\
&= -\int \frac{1}{4 - u^2}\,du \\
&= -\frac{1}{4}\ln\left|\frac{2+u}{2-u}\right| \\
&= -\frac{1}{4}\ln\left|\frac{2+(x-2)}{2-(x-2)}\right| \\
&= -\frac{1}{4}\ln\left|\frac{x}{4-x}\right|
\end{aligned}$$

Problem 83

Problem. Evaluate the definite integral $\displaystyle\int_3^7 \frac{1}{\sqrt{x^2 - 4}}\,dx$ using the formulas from Theorem 5.20.

Solution.

$$\begin{aligned}
\int_3^7 \frac{1}{\sqrt{x^2-4}}\,dx &= \Big[\ln\Big(x+\sqrt{x^2-4}\Big)\Big]_3^7 \\
&= \ln\Big(7+\sqrt{7^2-4}\Big) - \ln\Big(3+\sqrt{3^2-4}\Big) \\
&= \ln\Big(7+\sqrt{45}\Big) - \ln\Big(3+\sqrt{5}\Big) \\
&= \ln\Big(7+3\sqrt{5}\Big) - \ln\Big(3+\sqrt{5}\Big) \\
&= \ln\frac{7+3\sqrt{5}}{3+\sqrt{5}} \\
&= \ln\frac{3+\sqrt{5}}{2}.
\end{aligned}$$